The **mirror equation** is a geometrical derivation for solving problems with curved mirrors.

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}
\]

where:  
- \( f \) = focal length (in metres)  
- \( d_o \) = Object distance (in metres)  
- \( d_i \) = Image distance (in metres)

and

\[
M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}
\]

where:  
- \( M \) = magnification factor (no units)  
- \( h_i \) = height of image (in metres)  
- \( h_o \) = height of object (in metres)

Be sure to observe the following sign conventions when using this formula:  
1. All distances are measured from the vertex of a curved mirror.  
2. Distances of objects and real images are positive.  
3. Distances of virtual images are negative.  
4. Object and image heights are positive when upright and negative when inverted.

Parabolic mirrors are used as reflectors for searchlights, flashlights, projectors and automobile headlights.

1. An object is 60 cm from a **converging** mirror whose radius of curvature is 30 cm. Determine the characteristics of the image.

   \[
   d = \text{60 cm} \quad f = \text{15 cm}
   \]

   \[
   \therefore \quad \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}
   \]

   \[
   \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15\text{ cm}} - \frac{1}{60\text{ cm}} \quad \therefore \quad d_i = \text{20 cm}
   \]

   \[
   M = -\frac{d_i}{d_o} = -\left(\frac{20\text{ cm}}{60\text{ cm}}\right) = -0.33
   \]

   **Type:** Real
   **Location:** 20 cm in front of mirror
   **Magnification:** -0.33 (Reduced image)
   **Attitude:** Inverted

2. An object is 25 cm from a **concave** mirror whose focal length is 15 cm. Determine the characteristics of the image.

   \[
   d = \text{25 cm} \quad f = \text{15 cm}
   \]

   \[
   \therefore \quad \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}
   \]

   \[
   \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15\text{ cm}} - \frac{1}{25\text{ cm}} \quad \therefore \quad d_i = \text{37.5 cm}
   \]

   \[
   M = -\frac{d_i}{d_o} = -\left(\frac{37.5\text{ cm}}{25\text{ cm}}\right) = -1.5
   \]

   **Type:** Real
   **Location:** 37.5 cm in front of mirror
   **Magnification:** -1.5 (Magnified image)
   **Attitude:** Inverted
3. An object is placed 7.5 cm from a **concave** mirror that has a focal length of 15 cm. Determine the characteristics of the image.

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}
\]

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15} cm - \frac{1}{7.5} cm
\]

\[
M = \frac{-d_i}{d_o} = \frac{-(-15 cm)}{7.5 cm} = 2
\]

<table>
<thead>
<tr>
<th>Type:</th>
<th><strong>Virtual</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Location:</td>
<td>15 cm behind mirror</td>
</tr>
<tr>
<td>Magnification:</td>
<td>2 (Magnified image)</td>
</tr>
<tr>
<td>Attitude:</td>
<td>Upright</td>
</tr>
</tbody>
</table>

4. A **converging** mirror has a focal length of 15 cm. Where would you place an object in front of this mirror in order to produce an upright, virtual image, **twice** as tall as the object? [7.5 cm]

\[
f = 15 \text{ cm}
\]

\[
d_o = ?
\]

\[
h_i = 2h_o
\]

\[
M = 2
\]

\[
\therefore M = \frac{-d_i}{d_o} = \frac{-d_i}{M}
\]

or \( d_o = \frac{-d_i}{M} = \frac{-d_i}{2} \)

\[
\therefore d_i = -2d_o
\]

5. What is the focal length of a **concave** mirror when an object that is placed 2 cm in front of the mirror produces an image that is seen 5 cm behind the mirror? [3.3 cm]

\[
f = ?
\]

\[
d_o = 2 \text{ cm}
\]

\[
d_i = -5 \text{ cm}
\]

\[
\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}
\]

\[
= \frac{1}{2 \text{ cm}} + \frac{1}{-5 \text{ cm}}
\]

\[
= \frac{5 - 2}{10 \text{ cm}}
\]

\[
= \frac{3}{10 \text{ cm}}
\]

Thus: \( f = \frac{10 \text{ cm}}{3} = 3.3 \text{ cm} \)
6. A **concave** mirror has a focal length of 30 cm. A square object, 4 cm per edge, is situated with its center 10 cm in front of the mirror. Determine:

a) The position (center) of the image. [-15 cm]

\[ \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \]

or

\[ \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \]

\[ \frac{1}{30 \text{ cm}} = \frac{1}{10 \text{ cm}} \]

Thus: \( d_i = -15 \text{ cm} \)

b) The area of the image. [36 cm²]

\[ h_i = ? \]

\[ h_o = 4 \text{ cm} \]

\[ d_o = 10 \text{ cm} \]

\[ d_i = -15 \text{ cm} \]

Since: \( \frac{h_i}{h_o} = \frac{-d_i}{d_o} \)

Thus: \( h_i = \frac{-d_i h_o}{d_o} \)

\[ = \frac{-(-15 \text{ cm})(4 \text{ cm})}{10 \text{ cm}} \]

\[ = 6 \text{ cm} \]

Answer: \( 6 \text{ cm} \times 6 \text{ cm} = 36 \text{ cm}^2 \)

7. Illustrated below is an object situated in front of a **convex** mirror. Graphically, find the image and state its characteristics.

The image is virtual, upright, reduced and located behind the mirror.
8. An object is placed 10 cm in front of a convex mirror whose focal length is 15 cm. Determine the characteristics of the image.

\[
\begin{align*}
f &= 15 \text{ cm} \\
d_o &= 10 \text{ cm} \\
d_i &= ? \\
M &= ? \\
\frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\
M &= -\frac{d_i}{d_o} \\
\text{Thus:} \quad \frac{1}{d_i} &= \frac{1}{f} - \frac{1}{d_o} \\
&= \frac{1}{15 \text{ cm}} - \frac{1}{10 \text{ cm}} \\
&= 3 \\
d_i &= -30 \text{ cm}
\end{align*}
\]

The image is virtual (since \(d_i\) is negative), upright (since \(M\) is positive), three times larger than the object (since \(M = 3\)) and located behind mirror.

9. When an object is 30 cm in front of a concave mirror, its image is real and situated 15 cm from the mirror. Determine the location of the image if the object is placed 6.0 cm from the mirror. (15 cm)

\[
\begin{align*}
d_o &= 30 \text{ cm} \\
d_i &= ? \\
f &= 10 \text{ cm} \\
\text{Since:} \quad \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\
&= \frac{1}{30 \text{ cm}} + \frac{1}{15 \text{ cm}} \\
&= \frac{1}{10 \text{ cm}} - \frac{1}{6 \text{ cm}} \\
d_i &= -15 \text{ cm}
\end{align*}
\]

10. Fill in the four (4) missing quantities:

<table>
<thead>
<tr>
<th>MIRROR TYPE</th>
<th>FOCAL LENGTH</th>
<th>OBJECT DISTANCE</th>
<th>IMAGE DISTANCE</th>
<th>OBJECT HEIGHT</th>
<th>IMAGE HEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concave</td>
<td>4 cm</td>
<td>3 cm</td>
<td>-12 cm</td>
<td>2 cm</td>
<td>8 cm</td>
</tr>
<tr>
<td>Convex</td>
<td>23.3 cm</td>
<td>10 cm</td>
<td>-7 cm</td>
<td>1 cm</td>
<td>0.7 cm</td>
</tr>
</tbody>
</table>

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