

Nice Problem! Thanks for making my brain work for the first time in 24 hours.

Remember $P_A V_A = n_A RT$, so $[A] = n_A/V_A = P_A/RT$

Similarly $[B] = n_B/V_B = P_B/RT$

$[C] = n_C/V_C = P_C/RT$

$[D] = n_D/V_D = P_D/RT$

Hence,

$$K_C = \frac{[C]^c [D]^d}{[A]^a [B]^b} \text{ for } aA + bB = cC + dD$$

Substituting the first paragraph's equations into the above:

$$K_C = \frac{[P_C/RT]^c [P_D/RT]^d}{[P_A/RT]^a [P_B/RT]^b} = \frac{P_C P_D (RT)^a (RT)^b}{P_A P_B (RT)^c (RT)^d}$$

But $K_p = \frac{P_C P_D}{P_A P_B}$, so

$$K_C = K_p \frac{(RT)^a (RT)^b}{(RT)^c (RT)^d}$$

Isolate K_p :

$$K_p = K_C \frac{(RT)^c (RT)^d}{(RT)^a (RT)^b} = K_C \frac{(RT)^{c+d}}{(RT)^{a+b}}$$

$$K_p = K_C (RT)^{(c+d)-(a+b)}$$

$$K_p = K_C (RT)^{\Delta n}$$