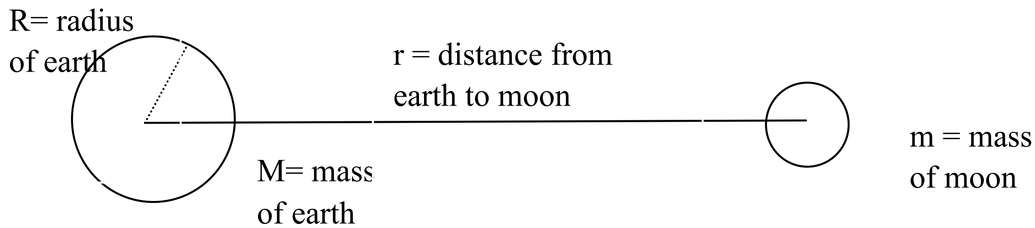
Use only g, π, the moon's period and earth's radius to calculate the distance to the moon.



Equate the force between the moon and the earth to the moon's centripetal force:

$$F = \frac{mv^2}{r} \text{ and } F = \frac{GmM}{r^2}$$

$$\frac{GmM}{r^2} = \frac{mv^2}{r}\text{; One r and m cancel:}$$

$$\frac{GM}{r} = v^2 \qquad \text{equation (1)}$$

Assuming that the moon's orbit is circular:

2πr = C The moon's velocity, v = C/T= 2πr/T, where T = its period.

Substituting for *v* in equation(1):

$$\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r^2}{T^2}$$

$$M = \frac{4\pi^2 r^3}{GT^2} \qquad \text{equation (2)}$$

On earth, the force of gravity on an object m_o is given by $m_o g$. The force between that object and the earth is also given by:

$$F = \frac{Gm_oM}{R^2}$$

$$m_o g = \frac{Gm_oM}{R^2}$$

$$g = \frac{GM}{R^2} \qquad \text{equation (3)}$$

Substitute equation (2) into equation (3):

$$g = \frac{G\frac{4\pi^2 r^3}{GT^2}}{R^2} = \frac{\frac{4\pi^2 r^3}{T^2}}{R^2} = \frac{4\pi^2 r^3}{T^2R^2} \quad \text{or} \quad r = \sqrt[3]{\frac{gT^2R^2}{4\pi^2}} = \sqrt[3]{\frac{9.8(27*24*3600+8*3600)^2(6.38X10^6)^2}{4\pi^2}}$$

= 3.8 X 10^8 m, pretty close to the mean distance between the earth and the moon.

The sun is 106.1 earth diameters wide. That means its radius is 106.1*6.38 X10^6 m = 6.76918 x 10^8 m. The earth-sun distance is 149 597 871 km = 1.49 597 871 10^{11} m.

Using the last equation we had for g but from the point of the sun we obtain:

$$g_s = \frac{4\pi^2 {r_{es}}^3}{T^2 {R_s}^2} = \frac{4\pi^2 (1.49\,597\,871\;10^{11})^3}{(365.25*24*3600)^2 (6.76918\;\mathrm{x}10^8)^2} = 289.6\ \mathrm{m/s^2}$$

With g_s we can now obtain the distance between the sun and any planet using its period of revolution. Let's use Jupiter as an example.

$$r = \sqrt[3]{\frac{g_s T^2 Rs^2}{4\pi^2}} = \sqrt[3]{\frac{289.6(374247820.8)^2 (6.76918\;\mathrm{x}10^8)^2}{4\pi^2}} = 7.779\ \mathrm{X}10^{11}\ \mathrm{m} = 7.779\ \mathrm{X}10^8\ \mathrm{km}$$

7.7857 X10^8 km is the accepted value

Go back to equation (2) and plug in r and G to get the mass of the earth: $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (3.8\ \mathrm{X}\ 10^8)^3}{6.673X10^{-11}(2361600)^2} = 5.8\ X10^{24}$ kg (accepted value = 5.9736 × 10^{24} kg)

In a simple two-body case, r_1, the distance from the center of the primary to the barycenter is given by:

Lunar Orbiter 3: Semimajor axis 2,694 km (1,674 mi) Eccentricity .33 Inclination 20.9° Apoapsis 1,847 km Period: =208.1 min

$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (\frac{2694X10^3}{2} + \frac{1847X10^3}{2})^3}{6.673X10^{-11}(208.1*60)^2} = 6.396\ \mathrm{X}\ 10^{22}$ kg (accepted value: 7.35 X 10^{22} kg)

where: $r_1 = \frac{a}{1+\frac{m_1}{m_2}}$

a is the distance between the centers of the two bodies;
m_1 and m_2 are the masses of the two bodies.