Derivation of Bohr's Radius and Rydberg's Constant

The centripetal force between the electron and the nucleus is balanced by the coulombic attraction between them:

$\frac{mv^2}{r} = \frac{Ze^2}{r^2}$, where Z = atomic number and e = charge of electron

Note that Coulomb's constant is not required if the unit $kg^{½}m^{3/2}s^{-1}$ is used for charge. Consequently, each expression will produce $kgms^{-2}$, which is a Newton. Alternately, once can express velocity and mass using cgs(cm-g-s) and then use the esu for charge.

1esu = 3.33564×10^{-10} C
1 $kg^{½}m^{3/2}s^{-1}$ = $1.05482 X10^{-5}$ C
Since -1C = 6.241506×10^{18} electrons, then
charge of e = -1.5189 9 X 10^{-14} $kg^{0.5}$ $m^{1.5}$ s^{-1}

$$mv^2 = \frac{Ze^2}{r}, \qquad (1)$$

$$\text{so } r = \frac{Ze^2}{mv^2} \qquad (2)$$

Since the electron's energy is characteristic of its orbit, the energy cannot be lost or gained. It is quantized. According to Bohr, the angular momentum of the electron is a whole number *n* multiple of $\frac{nh}{2\pi}$:

mvr = $\frac{nh}{2\pi}$, where v = velocity of electron; m = mass of electron; h = Planck's constant

$$v = \frac{nh}{2\pi mr} \qquad (3)$$

Substituting (3) into (2):

$$r = \frac{Ze^2}{m\left[\frac{nh}{2\pi mr}\right]^2} = \frac{Ze^2 4\pi^2 m^2 r^2}{mn^2h^2}$$

$$\frac{n^2h^2}{4\pi^2 mZe^2} = r$$

(4)

For the radius of hydrogen's atom
$Z = 1$
$n = 1$

$h = 6.6262 \text{ X } 10^{-34}$ Js = $6.6262 \text{ X } 10^{-34}$ kg m^2/s^2 (s) = $6.6262 \text{ X } 10^{-34}$ kg m^2/s
$m = 9.1096 \text{ X } 10^{-31}$ kg
$e = 1.5189\,9 \text{ X } 10^{-14}$ kg$^{0.5}$ m$^{1.5}$ s^{-1}

$$r = \frac{1^2\left(6.6262 \text{ X } 10^{-34} \text{ kg m}^2 \text{ s}^{-1}\right)^2}{4\pi^2(9.1096 \text{ X } 10^{-31} \text{ kg})(1)(-1.5189\,4 \text{ X } 10^{-14} \text{ kg}^{0.5} \text{ m}^{1.5} \text{ s}^{-1})^2} = 0.529 \text{ X}10^{-10} \text{ m}$$

= 0.529 Å

The total energy of the electron is the sum of its kinetic and potential energies:

$E_T = E_k + E_p$

$= mv^2/2 - Ze^2/r$ (5)

Substituting (1) into (5):

$E_T = Ze^2/2r - Ze^2/r$

$E_T = Ze^2/2r - 2Ze^2/2r$

$E_T = -Ze^2/2r$ (6)

Substituting (4) into (6):

$$E_T = \frac{-Ze^2}{2\left(\dfrac{n^2h^2}{4\pi^2 mZe^2}\right)} = \frac{-4\pi^2 mZ^2e^4}{2n^2h^2} = \frac{-2\pi^2 mZ^2e^4}{{n_o}^2h^2}$$

When an electron falls back to a lower energy level, it emits a photon of energy hυ, which is the difference in the energy between the energy outer level, E_o and that of the inner level, E_i

$$h\,\upsilon = E_o - E_i = \left(\frac{-2\pi^2 mZ^2e^4}{{n_o}^2h^2} - \frac{-2\pi^2 mZ^2e^4}{{n_i}^2h^2}\right) = \frac{2\pi^2 mZ^2e^4}{h^2}\left(\frac{1}{{n_i}^2} - \frac{1}{{n_o}^2}\right)$$

$$\upsilon = \frac{2\pi^2 mZ^2e^4}{h^3}\left(\frac{1}{{n_i}^2} - \frac{1}{{n_o}^2}\right), \text{ where}$$

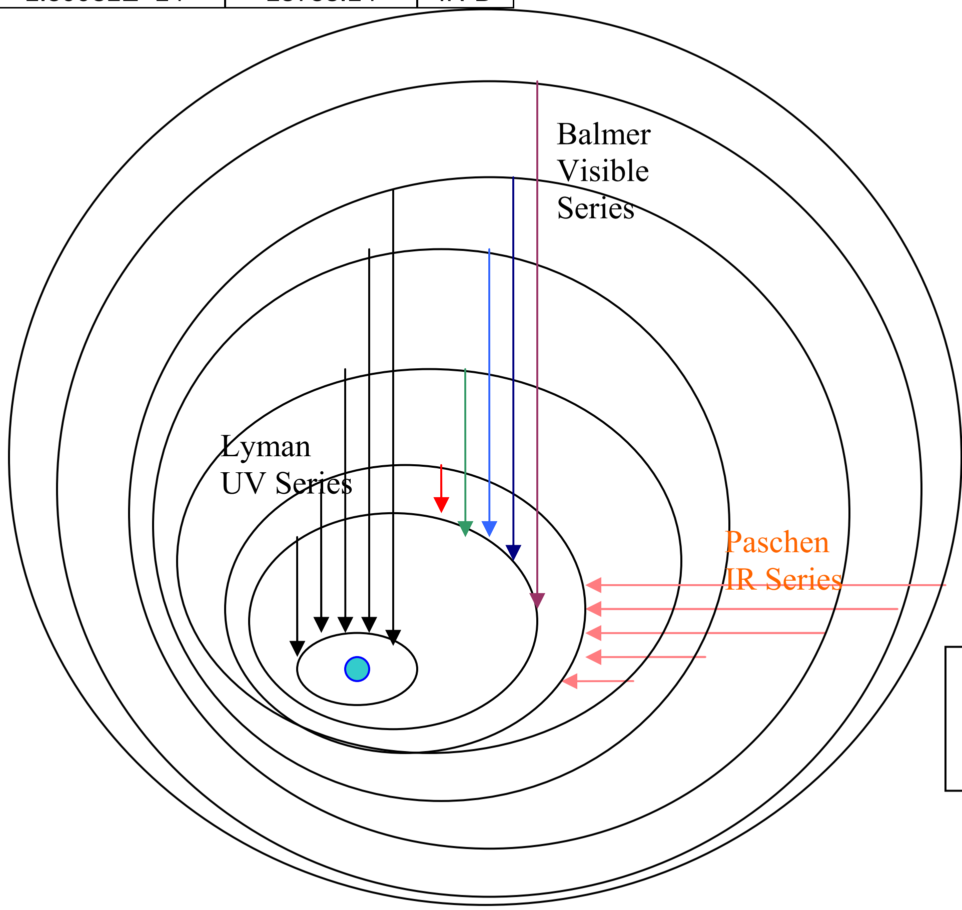
$$\frac{2\pi^2 mZ^2e^4}{h^3} = \frac{2\pi^2(9.1096 \text{ X } 10^{-31}\text{kg})(1)^2(-1.51894 X10^{-14} kg^{0.5}m^{1.5}s^{-1})^4}{(6.6262\, X10^{-34}\, kg\, m^2 s^{-1})^3} = 3.29 \text{ X } 10^{15} \text{ s}^{-1}$$

This is known as Rydberg's constant and it can be used to calculate the frequencies for various emissions of hydrogen.

n_o =N outer	**Lyman ν (s^{-1})** n_i= n inner = 1	**Lyman wavel.(angstroms) λ =c/ν**	**radiation type**	**Balmer ν (s^{-1})** n_i = 2	**Balmer λ (angstroms)**	
7				7.55357E+14	3971.631	violet
6	3.19861E+15	937.9070777	EUV	7.31111E+14	4103.343	blue
5	3.1584E+15	949.8480243	EUV	6.909E+14	4342.162	blue
4	3.08438E+15	972.6443769	EUV	6.16875E+14	4863.222	green
3	2.92444E+15	1025.835866	UVC	4.56944E+14	6565.35	red
2	2.4675E+15	1215.805471	UVC			

EUV = extreme ultravilolet
UVC = ultraviolet C

Paschen ν (s-1) n_i = 3	**Paschen λ (angstroms)**	**type**
n_i = 3		
2.98413E+14	10053.19	IR-A
2.74167E+14	10942.25	IR-A
2.33956E+14	12822.95	IR-A
1.59931E+14	18758.14	IR-B



Energy Levels and Transitions in Hydrogen Atom

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