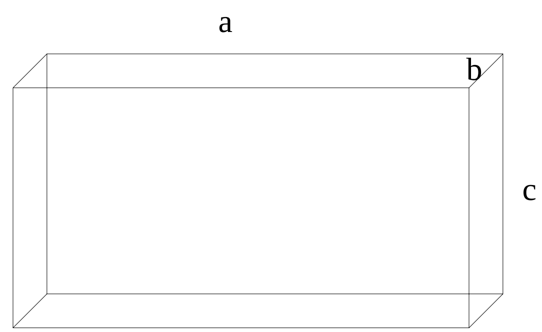
In Isaac Asimov's Understanding Physics Volume I, George Allen and Win, London (1966), there is a simple derivation of the relationship between temperature and kinetic energy. Here is my slightly extended version of it, and if you compare Asimov's approach to that found in a typical first year university physical chemistry textbook, you appreciate how good a teacher the late Asimov was.



Consider a box of dimensions a, b,and c filled with N particles, each with a mass m, moving with a velocity v. The particles are free to move in any direction, but any motion can be decomposed into three components, x y and z, perpendicular to one other. That means that 1/3 of all particles will have a component moving along one edge, say a. (Collisions between particles can be ignored because they are perfectly elastic and because the direction-change caused by a collision will be cancelled by another collision causing a change in the opposite direction.)

If a particle hits the wall represented by edge bc, it will experience a change in momentum 2mv. [= mv - (-mv), since the particle will bounce off bc in the opposite direction]

To get the total force exerted on the wall bc, we need to know how many collisions occur on that wall per unit time. (Recall that impulse(Force*time) equal momentum). Before the next collision with wall bc, the particle must travel a distance of 2a. Since it moves with velocity v, by dividing v by 2a, we get the frequency or the number of collision per second. Confusing? Suppose that a = 10 m, for a total length of 2a = 20 m. If the particle moved at 5 m/s, it would have a collision frequency of v/2a= 5/20= 0.25/s equivalent to 1 collision every 4 seconds.

The total force exerted on the wall by one particle in one second equals the change in momentum in one bounce times the collision frequency:

2mv(v/[2a]) = mv^2/a. To get the force exerted by all N particles, remember that only one third of the particles are moving that way, so F = $Nmv^2/(3a)$. To get an expression for pressure, which is force per unit area, we simply divide F by the area of the wall in question, yielding

$$P = \frac{F}{bc} = \frac{Nmv^2}{3abc}.$$

Since abc = volume of the box,

$$P = \frac{Nmv^2}{3V}.$$

Multiplying both the numerator and denominator by 2:

$$P = \frac{2(Nmv^2)}{2(3V)} = \frac{2NE_k}{3V}$$

, since kinetic energy, Ek = $mv^2/2$.

Rearranging PV = (2/3)N Ek. For one mole of particles, N = n, (correspondingly, we would have m becoming the molar mass)so

PV = (2/3)n Ek

Since PV = nRT,

Equating the two above equations:

RT = (2/3) Ek

T = (2/3R) Ek, or

Ek, =(3/2)RT.

Or $mv^2/2$ = (3/2)RT.

$$v = \sqrt{\frac{3RT}{m}},$$

where m = molar mass in kg

Example: At 273 K, what is the average velocity of oxygen molecules?

v =

$$\sqrt{\frac{3(8.31)273}{0.032}} = 461 \text{ m/s}$$

A dimensional analysis for the above problem(to check the units):

1000 L = 1 m3. 1kPa = 1000 N/m2

R = 8.31 L*kPa/(K *mole) 1 N = 1 kg*m/s2

$$v = \sqrt{\frac{3(8.31 \frac{m^3 \frac{N}{m^2}}{K\ mole})(273\ K)}{0.032\ \ kg/mole}} = \sqrt{212684.0625 \frac{\mathrm{Nm}}{\mathrm{kg}}} = \sqrt{212684.0625 \frac{\mathrm{kg(\ m/s^2\ )m}}{\mathrm{kg}}} = 461\ m/s$$