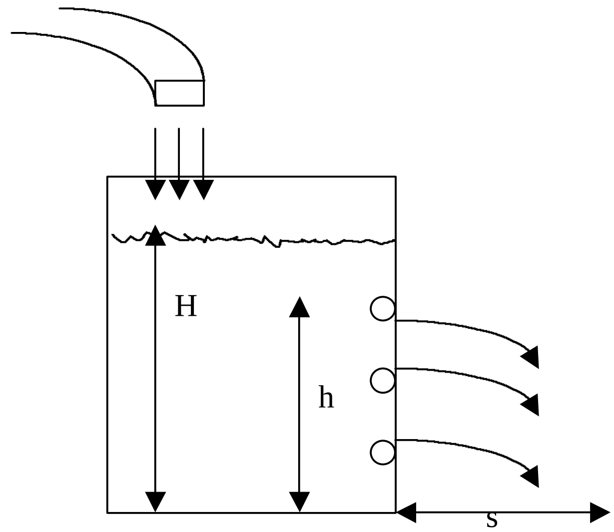
The Conservation of Energy and Projectile Motion



This problem is from the book of puzzles *Mad About Physics by Jargodzski and Potter. Wiley & sons. 2001*. To fully appreciate it:

(1) make a prediction with some rationale
(2) *try it* to see what really happens
(3) understand what happened.

Three holes are drilled into a metal can the size of a 1 kg package of infant formula. The first hole is halfway between the bottom of the can and the water level, which is kept constant by adjusting the flow from a faucet. The other two holes are, respectively, ¼ and ¾ of the distance from the can's base.

The question is: predict how far the water will fall from each hole. In other words, predict *s* (see diagram) for each hole.

The solution is on the next 2 pages.

Solution

One would expect that the water near the lowest exit point is under the most pressure, so that it should travel the furthest. But it does not. The picture reveals that the water from the middle-hole is traveling about a centimeter past the farthest reaches of the other two projectiles, which are practically identical.

Here's why. Since energy is conserved, the sum of potential and kinetic energy is identical for the water at any hole. As you move lower from the water level, potential energy decreases but kinetic energy increases. More specifically, the kinetic energy gained will equal the loss in potential energy , which is caused by the decrease in height.

$\frac{1}{2} mv^2 = mg\Delta h$
or $v^2 = 2g\Delta h$
$v = \sqrt{2g\Delta h}$ (1)

So far this seems to reinforce the incorrect prediction that water from the lowest hole will travel the longest distance since it does start with the highest horizontal velocity. But the water is also in freefall and the time that it takes for the water to fall will depend on its initial height.

$h = ½ gt^2$

$$t = \sqrt{\frac{2h}{g}} \qquad (2)$$

Since the horizontal distance traveled by the water is the product of horizontal velocity and time

s = vt.

Substituting the expressions for velocity and time from equations (1) and (2):

$$s = \sqrt{2g\Delta h}\left[\sqrt{\frac{2h}{g}}\right] = 2\sqrt{h\Delta h} = 2\sqrt{h(H-h)}$$

Whether h = ¾ H or ¼ H, s =$2\sqrt{\frac{3H^2}{16}} = \frac{H\sqrt{3}}{2}$. In my trial I used a water level of 12 cm, so for the first and third holes the horizontal distance was about 10.4 cm in each case.

But for the middle hole, h =1/2 H , so s = $2\sqrt{\frac{H}{2}(H-\frac{H}{2})}$ = H. Looking at the picture you see an approximately 1 cm difference (theoretical : 12 – 10.4 cm = 1.6 cm) between the middle hole's projectile and the rest.