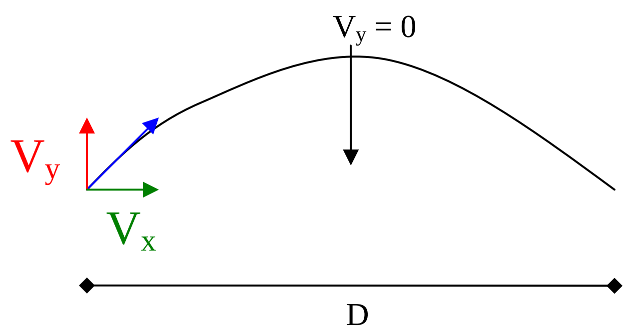
Something projected at an angle has two components, a vertical component and a horizontal one. If its initial velocity is $v_0$ then its vertical component is $v_y$ and its horizontal component is $v_x$.




Unless there is friction(an external force), the horizontal component will stay constant(Newton's first)law. What creates the curvature is the fact that the overall velocity is the constant resultant of two components, one of which is constantly changing.

The vertical component will keep shrinking because of gravity until the projected seed reaches its maximum height. At this point(see diagram) $v_y = 0$, but immediately after it starts to increase again until it reaches its maximum value again as it hits the ground.

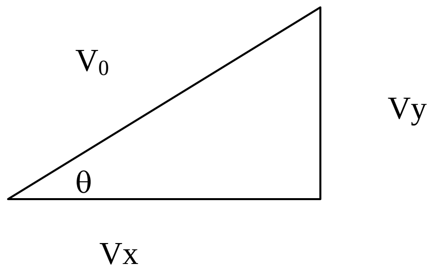
That implies that the time($t_1$) it takes the seed to go from its maximum height to the ground would be equal to the time it would take the seed to drop straight down from the maximum height, so

$$V_y = V_{yi} + gt_1, \text{ but } V_y \text{ (initial)} = 0, \text{ so}$$

$$V_y = gt_1 \text{ or}$$

$$t_1 = \frac{v_y}{g}$$

We can express $v_y$ in terms of $v_0$(initial velocity) using trigonometry:




$\sin\theta = \frac{V_y}{V_0}$ Note that $\cos\theta = \frac{V_x}{V_0}$

So $V_y = V_0 \sin\theta$

Substituting into previous expression for time:

$$t_1 = \frac{V_0 \sin\theta}{g}$$

the total time of flight(it has to go up and back down): $t = 2t_1$.

The horizontal distance(D) traveled is simply the constant horizontal velocity times total time:

$D = v_x(t) = v_x(2t_1)$

Substituting for $t_1$,

$D = v_x\, 2\frac{V_0 \sin\theta}{g}$, but $\cos\theta = \frac{V_x}{V_0}$,and $V_x = V_0\cos\theta$, so

$$D = \frac{V_0^{\,2}\, 2\sin\theta\cos\theta}{g}$$

Since $2\sin\theta\cos\theta = \sin 2\theta$, then

$D = \frac{V_0^{\,2}\sin 2\theta}{g}$, and if $\theta = 45^{o}$, then

$D = \frac{V_0^{\,2}}{g}$ or

$$V_o = \sqrt{gD}$$

***Without friction*** we would expect a distance of 15 m laterally to correspond to an initial velocity of only Vo = $\sqrt{9.8 m/s^2 (15 m)} = 12.1 m/s$

12.1 m/s(3600 s/h)(1 km/1000m) = 43.6 km/h