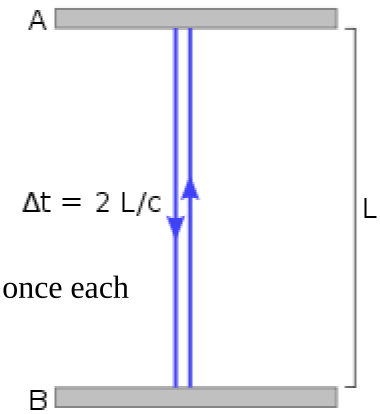
This is basically copied from Wikipedia's article on time dilation, but what I've done is filled in many of the basic algebraic steps that the article skips over, and I added the time dilation factor , γ, which was omitted by the original author.



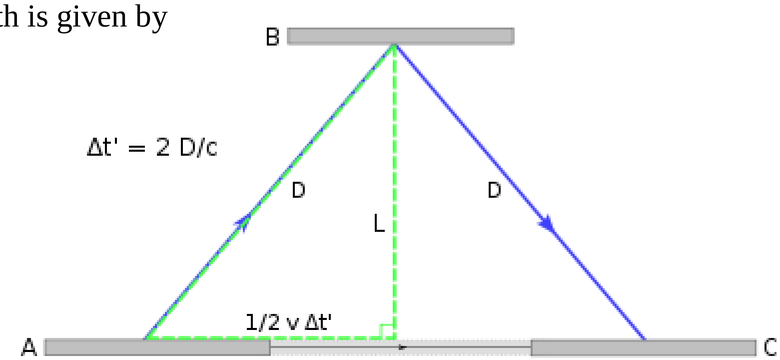
Consider a simple clock consisting of two mirrors A and B, between which a light pulse is bouncing. The separation of the mirrors is **L**, and the clock ticks once each time it hits a given mirror.

In the frame where the clock is at rest (diagram at right), the light pulse traces out a path of length 2**L** and the period of the clock is 2**L** divided by the speed of light:

$$\Delta t = \frac{2L}{c}.$$

From the frame of reference of a moving observer traveling at the speed *v* (diagram at lower right), the light pulse traces out a *longer*, angled path. The second postulate of special relativity states that the speed of light is constant in all frames, which implies a lengthening of the period of this clock from the moving observer's perspective. That is to say, in a frame moving relative to the clock, the clock appears to be running more slowly. Straightforward application of the Pythagorean theorem leads to the well-known prediction of special relativity:

The total time for the light pulse to trace its path is given by



$$\Delta t' = \frac{2D}{c}.$$

Or $D = \frac{c\Delta t'}{2}$

The length of the half path can be calculated as a function of known quantities as

$$D^2 = \frac{v^2 \Delta t'^2}{4} + L^2$$

Substituting ***D*** from this equation into the previous,

$$\frac{c^2\Delta t'^2}{4} = \frac{v^2\Delta t'^2}{4} + L^2$$

$$or\ multiplying\ through\ by\ 4\ and\ solving\ for\ \Delta t':$$

$$c^2\Delta t'^2 = v^2\Delta t'^2 + 4L^2$$

$$c^2\Delta t'^2 - v^2\Delta t'^2 = 4L^2$$

$$\Delta t'^2(c^2 - v^2) = 4L^2$$

$$\Delta t'^2 = \frac{4L^2}{(c^2 - v^2)}$$

Multiply numerator and denominator by $1/c^2$

$$\Delta t'^2 = \frac{(\frac{1}{c^2})4L^2}{(\frac{1}{c^2})(c^2 - v^2)}$$

$$\Delta t'^2 = \frac{4L^2/c^2}{(1 - \frac{v^2}{c^2})}$$

$$\Delta t' = \frac{2L/c}{\sqrt{1 - v^2/c^2}}$$

and thus, with the definition of $\boldsymbol{\Delta t}$:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}$$

which expresses the fact that for the moving observer the period of the clock is longer than in the frame of the clock itself.

If you want to express the time dilation as a factor, then,

$$\gamma = \frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1 - v^2/c^2}}$$